Fac. Sc. Ain Chock, Casablanca

Topological Robotics

Rational Topological Complexity

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Summary

بِسمِ اللَّهِ الرَّحمَنِ الرَّحِيمِ

وَ عَلَى اللَّهِ فَلْيَتَوَكَّلِ المُتَوَكِّلُون

صَدَقَ اللَّهُ العَظِيم

1. Sectional Category : Prerequisite
2. Rationalization

prerequisite

sectional category of a fibration

Definition

$\mathrm{secat}(\mathrm{p} : \mathrm{E} \longrightarrow \mathrm{B})$, is the least integer n such that the base space B can be covered by $n+1$ open subspaces on each of which p admits a section. If no such n exists one sets secat = ∞

Homotopy invariant

introduced by Albert Schwarz (1934,–), Univ. Califronia Davis, USA) in the late 1950's as a generalization of the Lusternik–Schnirelmann category of a space.

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sectional category of a fibration

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prerequisite

Lusternik-Schnirelmann category

Definition

For any topological space, *cat*(X) is the smallest n for which there is an open covering $\{U_0, \ldots, U_n\}$ by $(n+1)$ open sets, each of which is contractible in X.

Poland (1899-1981)

Russia (1905-1938)

prerequisite

secat vs. LS-cat

Remark

If X is a path connected space with base point x_0 and PX is the space of paths beginning at x_0 then

$$cat(X) = secat(ev_1)$$

where

$$\begin{array}{cccc} ev_1 & PX & \longrightarrow & X \\ & \gamma & \longmapsto & \gamma(1) \end{array}$$

prerequisite

secat vs. complexity

Secat

measures the complexity of certain algorithms, like that of finding roots for algebraic equations (S. Smale, (1930, –), Univ. California, Berekely, USA).

prerequisite

secat vs. topological complexity

M. Farber, Univ. Warwick, England

$TC(X) = secat(ev_{0,1})$ where

$$\begin{array}{rccc} ev_{0,1}: & PX & \longrightarrow & X \times X \\ & \gamma & \longmapsto & (\gamma(0), \gamma(1)) \end{array}$$

TC plays an important role in robotics to measure the complexity of finding a motion planning algorithm.

prerequisite

secat vs. nilpotency

nilker

Let $p^* : H^*(B) \longrightarrow H^*(E)$, then *nil* $\ker p^*$ is the least integer n such that any (n+1)-fold cup product in $\ker p^*$ is trivial.

Propriety

$$\ker p^* \leq secat_0(p) \leq secat(p)$$

prerequisite

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$$\ker p^* \leq secat_0(p) \leq secat(p)$$

Rationalization

inspiring question

M. Farber (2006)

Is there any algebraic description of the rational version of TC(X) in terms of the Sullivan minimal model of X ?

In all the remainder of this talk : X is a simply connected CW-complex of finite type, and $(\Lambda V, d)$ will denoted its minimal model and $X_{\mathbb{Q}}$ its rationalization.

Rationalization

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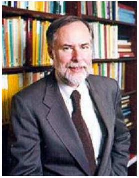
Rationalization

inspiring idea

S. Halperin & J.M. Lemaire (88)

$Mcat(X)$ is the least integer m such that the inclusion in $(\Lambda V, d) \hookrightarrow (\Lambda V / \Lambda^{>m} V, d)$ admits a retraction of differential graded ΛV-modules.



Steve Halperin (1945, –)

Rationalization

inspiring result

Kathryn Hess (91)

$$cat(X_{\mathbb{Q}}) = Mcat(X)$$

K. Hess (1967, –)

Rationalization

First generalisation attempt

A. Fasso Velenik

In her thesis [2002-Berlin], she gave a characterization of $secat_0(p)$ in terms of a Sullivan model of p. Unfortunately, concrete computations based on this characterization turn out to be rather difficult due to the complexity of the algebraic manipulations involved.

Rationalization

The Join concept

- Let $p : E \longrightarrow B$ and $p' : E' \longrightarrow B$ two maps.
- **Double mapping cylinder :**

  $E *_B E' := ((E \times_B E') \times I \coprod E \coprod E')/ \sim$ where

  $$(e, e', 0) \sim e; (e, e', 1) \sim e'$$

- **Join map :** $j_{p,p'} : E *_B E' \longrightarrow B$ where

  $$j_{p,p'}[e, e', t] = p(e) = p'(e'), j_{p,p'}[e] = p(e), j_{p,p'}[e'] = p'(e')$$

- ***n*-fold join :** $*_B^0 E := E, *_B^n E := (*_B^{n-1} E) *_B E.$
- ***n*th join map :** $j^0 p := p, j^n p := j_{j^{n-1} p, p'}.$

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Rationalization

The care of simplicity

Lucile Vandembroucq and all(2006)

Consider the morphism $A_{PL}(j^n p) : A_{PL}(B) \longrightarrow A_{PL}(*^n_B E)$ as a . The invariant $Msecat(p)$ is defined to be the least integer n for which $A_{PL}(j^n p) = \phi \circ i$ where ϕ is a quasi-isomorphism and i admits a retraction. (morphisms are of $A_{PL}(B)$-modules)

L. Vandembroucq (1973,

Rationalization

First results :

Lucile Vandembroucq and all(2006)

1. $nil\ker p^* \leq Msecat(p) \leq secatp$ when the base is normal ;
2. $Msecat(ev_1) = Mcat(X)$ when X is simply connected of finite type ;
3. $MTC(X) := Msecat(ev_{0,1})$ is an invariant of the rational homotopy type ;
4. $MTC(X) \leq TC(X_{\mathbb{Q}}) \leq TC(X)$;
5. $nil\ker \cup \leq MTC(X) \leq nil\ker \mu_{\Lambda V}$;
6. $nil\ker \cup = MTC(X)$ when X is formal ;
7. $Mcat(X) \leq MTC(X) \leq 2Mcat(X)$.

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Rationalization

First minimal model description :

Lucile Vandembroucq and all(2006)

Mseact is the least n such that the inclusion

$$(\Lambda U, d) \hookrightarrow *^{n}_{(\Lambda U, d)}(\Lambda W, d)$$

admits a retraction of $(\Lambda U, d)$-modules. Where $(\Lambda U, d)$ is a minimal model of the base B and $(\Lambda W, d)$ that of E.

Rationalization

Decisive step :

L. Lechuga and A. Murillo (2007)

They give a characterization of $TC(X_{\mathbb{Q}})$ in terms of a Sullivan model. Unfortunately, concrete computations based on this characterization turn out to be rather difficult due to the complexity of the algebraic manipulations involved.

A. Murillo (1965, –)

Rationalization

Main result

B. Jessup, A. Murillo and P. E. Parent (2012)

$TC(X_{\mathbb{Q}})$ is the smallest n for which the projection

$$\Lambda V \otimes \Lambda V \longrightarrow \Lambda V \otimes \Lambda V \diagup (\ker \mu)^{n+1}$$

admits a homotopy retraction.
Where μ is the multiplication

$$\mu : \Lambda V \otimes \Lambda V \longrightarrow \Lambda V .$$

B. Jessup (1963, –)

Rationalization

Main result

Inspiration

This description was inspired directly by Y. Félix and S. Halperin. In 1982 they first gave a useful and highly successful algebraic characterization of the Lusternik-Schnirelmann category of X_Q in terms of Sullivan minimal models.

Y. Félix (1951, –)

Rationalization

Main Result

Main consequences

1. $MTC(X)$ is the smallest n for which the projection $\Lambda V \otimes \Lambda V \longrightarrow \Lambda V \otimes \Lambda V \big/ (\ker \mu)^{n+1}$ admits a homotopy retraction as $\Lambda V \otimes \Lambda V$-modules.
2. $MTC(X) \leq TC(X_{\mathbb{Q}})$.
3. $MTC(X \times \mathbb{S}^n) = MTC(X) + MTC(\mathbb{S}^n)$.
4. $TC(X \times \mathbb{S}^n) = TC(X) + TC(\mathbb{S}^n)$. when X is formal and rational.
5. $nil \ker \cup_{\mathbb{Q}} \leq MTC(X) \leq nil \ker \mu_{\Lambda V}$
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Rationalization

One application

Hard Problem ! !

Due to the well known inequality

$$cat(X) \leq TC(X) \leq 2cat(X)$$

and the fact that determining the finiteness of the LS-category of a rational space is NP-hard. The simple characterization of TC with minimal models is particularly is interesting in view of the fact that determining whether or not the topological complexity of a rational space is finite, is an NP-hard problem.

Rationalization

Last attempt : a care of generalisation

J.G. CARRASQUEL VERA (2012)

Let $f : X \longrightarrow Y$ a map and $f_0 : X_{\mathbb{Q}} \longrightarrow Y_{\mathbb{Q}}$ its rationalization and $(\Lambda V, d) \longrightarrow (\Lambda W, d)$ a surjective model of f_0 with kernel K. Let n be the least integer such that the quotient map

$$(\Lambda V, d) \longrightarrow (\Lambda V \diagup K^{n+1}, d)$$

admits a homotopy retraction, then

$$n \geq secat(f_0)$$

Rationalization

Recalling : Higher TC

Y. Rudyak (2012)

$$TC_n(X) = secat(\Delta : X \longrightarrow X^n) = secat\left(ev_{0,\frac{1}{n-1},\dots,1}\right)$$

where

$$\begin{array}{cccc} ev_{0,\dots,1} & PX & \longrightarrow & X^n \\ & \gamma & \longmapsto & (\gamma(0), \gamma(\frac{1}{n-1}), \dots, \gamma(1)) \end{array}$$

Y. Rudyak (1948, –)

Recall that $TC_2 = TC$

Rationalization

Recalling : Higher TC

Y. Rudyak (2012)

$$
\begin{aligned}
TC_n(X) &= secat(\Delta : X \longrightarrow X^n) \\
&= secat\left(ev_{0,\frac{1}{n-1},\ldots,1}\right)
\end{aligned}
$$

where

$$
\begin{array}{cccc}
ev_{0,\ldots,1} & PX & \longrightarrow & X^n \\
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Y. Rudyak (1948, –)

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Rationalization

The care of generalisation : Main results

J.G. CARRASQUEL VERA (2012)

1. If X is formal, then :
   - $$TC_n(X_{\mathbb{Q}}) = nil \ker(\Delta^* : H^*(X;\mathbb{Q}) \longrightarrow H^*(X^n;\mathbb{Q})) \; ; \quad = nil \ker(\mu_n : H^*(X;\mathbb{Q})^{\otimes n} \longrightarrow H^*(X;\mathbb{Q}))$$
   - $TC_n(X_{\mathbb{Q}}) = TC(X_{\mathbb{Q}}) + (n-2)cat(X_{\mathbb{Q}})$;
2. $TC_n(X_{\mathbb{Q}}) = (n-1)cat(X_{\mathbb{Q}}) = (n-1)\dim V$ when V is only odd graded.

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J.G. CARRASQUEL VERA (2012)

1. If X is formal, then :
   - $$\begin{aligned} TC_n(X_{\mathbb{Q}}) &= nil \ker(\Delta^* : H^*(X;\mathbb{Q}) \longrightarrow H^*(X^n;\mathbb{Q}) \quad ; \\ &= nil \ker(\mu_n : H^*(X;\mathbb{Q})^{\otimes n} \longrightarrow H^*(X;\mathbb{Q}) \end{aligned}$$
   - $TC_n(X_{\mathbb{Q}}) = TC(X_{\mathbb{Q}}) + (n-2)cat(X_{\mathbb{Q}})$;
2. $TC_n(X_{\mathbb{Q}}) = (n-1)cat(X_{\mathbb{Q}}) = (n-1)\dim V$ when V is only odd graded.

Rationalization

The care of generalisation : Main results

J.G. CARRASQUEL VERA (2012)

1. If X is formal, then :
   - $$TC_n(X_{\mathbb{Q}}) = nil\ker(\Delta^* : H^*(X;\mathbb{Q}) \longrightarrow H^*(X^n;\mathbb{Q}) \quad ;$$
     $$= nil\ker(\mu_n : H^*(X;\mathbb{Q})^{\otimes n} \longrightarrow H^*(X;\mathbb{Q})$$
   - $TC_n(X_{\mathbb{Q}}) = TC(X_{\mathbb{Q}}) + (n-2)cat(X_{\mathbb{Q}})$;
2. $TC_n(X_{\mathbb{Q}}) = (n-1)cat(X_{\mathbb{Q}}) = (n-1)\dim V$ when V is only odd graded.

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That's all talks