بِسمِ اللَّهِ الرَّحمَنِ الرَّحِيمِ

وَ عَلَى اللَّهِ فَليَتَوَكَّلِ المُتَوَكِّلون

صَدَقَ اللَّهُ العَظِيم

The conjecture (H):

A conjectured lower bound for the cohomological dimension of elliptic spaces.

My Ismail Mamouni (Doctoral)

University Hassan II, Casablanca, Morocco.

Third Arolla Conference on Algebraic Topology
18-24 August 2008
Hôtel Mont-Collon, Arolla, Switzerland

What about me?

- I am an agregate professor "professeur agrégé" in math. teatching maths in Preparatory Classes of Engeineers Schools, Morocco since 1995
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Field of my resarch.

Rational Homotopy Theory

Subject of my work.

In my research, I am interersted to the following conjecture:

The sum of the Betti numbers of a 1-connected elliptic space is greater than the total rank of its homotopy groups.

The conjecture H

The conjecture H: Topological version

If X is a 1-connected elliptic space then

$$\dim H^*(X,\mathbb{Q}) \geq \dim(\pi_*(X)\otimes\mathbb{Q})$$

Let us recall that a space X is called elliptic when both of $H^*(X,\mathbb{Q})$ and $\pi_*(X)\otimes\mathbb{Q}$ are finite-dimensional.

The conjecture H was formulated in 1990, by my advisor professor Mohamed Rachid Hilali in his thesis with Yves Felix (Univ. Louvain, Belgium) .

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The tool: Models of Sullivan

Our main tool to model the conjecture H in an algebraic version is the notion of Sullivan's models

For any 1-connected space X of finite type, i.e., $\dim H^k(X,\mathbb{Q}) < \infty$ for all $k \geq 0$, Denis Sullivan associated a commutative differential graded algebra $(\Lambda V,d)$ (unique up to isomorphism) , called minimal Sullivan model of X, which algebraically models the rational homoptopy type of the space. More precisely

$$H^*(\Lambda V,d) \cong H^*(X,\mathbb{Q}) \quad \text{as algebras}$$
$$V \cong \pi_*(X) \otimes \mathbb{Q} \quad \text{as vector spaces}$$

Thus a space X is called elliptic when its model does, i.e., V and $H^*(\Lambda V,d)$ are both finite-dimensional.

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In his thesis in 1990, M.R. Hilali has shown that the conjecture H holds for *pure spaces*; these are spaces for which the model verifies the propretie

$$\begin{cases} dV^{\text{even}} = 0 \\ dV^{\text{odd}} \subset V^{\text{even}} \end{cases}$$

In 2006, we solved the conjecture H in the following cases:

- For H-spaces.
- For the so-called hyperelliptic spaces, under the additional condition that $\chi_c \geq \frac{1}{2}(1+\sqrt{-12\chi_\pi - 15})$

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- For 1-connected and hyperelliptic spaces with the additional condition that $-2 \leq rk_0(X) + \chi_\pi \leq 0$
  $rk_0(X)$: The toral rank of a space X is the maximum of n such that the toric $\mathbb{T}^n$ acts almost freely on X.
- For 1-connected and elliptic spaces with the additional condition that $0 \leq fd(X) - rk_0(X) \leq 6$.
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In 2008, we were interested to some works of Barry Jessup and Gregory Lupton on the homogeneous lenght of the differential.

Definition

- We say that ΛV has differential d of homegeneous-length k if $dV \subset \Lambda^k V$.
- We say that ΛV has differential d of homegeneous-length at least k if $dV \subset \Lambda^{\geq k} V$.

Using these works, we have resolved the conjecture H in the following cases:

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Using these works, we have resolved the conjecture H in the following cases:

- If $(\Lambda V, d)$ is an elliptic minimal model with an homogeneous-length differential and whose rational Hurewicz homorphism is non-zero in some odd degree.
- If $(\Lambda V, d)$ is an elliptic minimal model with an homogeneous-length differential at least 3.
- If $(\Lambda V, d)$ is an elliptic minimal model with a differential, homogeneous of length 2 (i.e: coformal) with odd degree generators only, (i.e., $V^{\text{even}} = 0$)

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Finally, our last and most important result is that we have shown that the conjecture H holds for formal spaces.

Theorem

If X is a 1-connected and formal elliptic space, then

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Steps of the proof.

- We know form [?] that X admits a model $(\Lambda V, d)$ of the form

$$V = V_0 \oplus V_1$$

with

$$V_0^{\text{even}} = \bigoplus_{1 \leq i \leq n} \mathbb{Q} x_i$$
$$V_0^{\text{odd}} = \bigoplus_{1 \leq i \leq q} \mathbb{Q} z_i$$
$$V_1 = V_1^{\text{odd}} = \bigoplus_{1 \leq j \leq m} \mathbb{Q} y_j$$
$$dV_0 = 0$$
$$dy_j = w_j \in \Lambda V_0$$

- Let $(\Lambda V, d_\sigma)$ denotes the pur model associated to (Λ, V), (see [?]-page 438).
- (Proposition 32.4, [?], page 438) states that $\dim H^*(\Lambda V, d_\sigma) < \infty$.
- Write $w_j = \alpha_j + \beta_j$ where $\alpha_j \in \Lambda V_0^{\text{even}}$ and $\beta_j \in \Lambda^+ V_0^{\text{odd}}.\Lambda V_0$, then $d_\sigma y_j = \alpha_j$.
- $H^*(\Lambda V, d_\sigma) = \dfrac{\Lambda(x_1, \cdots, x_n)}{(\alpha_1, \cdots, \alpha_m)} \otimes \Lambda(z_1, \cdots, z_q)$
- $m = n$ and then $\dim V = 2n + q$.
- Let $W_0 = H_0(\Lambda, V), W_1 = \langle [x_i], i = 1..n\rangle$ and $W_1 = \langle [x_i x_j], i,j = 1..n\rangle$.

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- $H^*(\Lambda V, d_\sigma) = \dfrac{\Lambda(x_1, \cdots, x_n)}{(\alpha_1, \cdots, \alpha_m)} \otimes \Lambda(z_1, \cdots, z_q)$
- $m = n$ and then $\dim V = 2n + q$.
- Let $W_0 = H_0(\Lambda, V), W_1 = \langle [x_i], i = 1..n \rangle$ and $W_1 = \langle [x_i x_j], i, j = 1..n \rangle$.

- The minimalty of the model $(dV \subset \Lambda^{\geq 2} V)$ insures us that $W_0 \oplus W_1 \oplus W_2 \oplus V_0^{\text{odd}}$ is a direct sum in $H^*(\Lambda V, d)$.
- $\dim W_0 = 1, \dim W_1 = n, \dim V_0^{\text{odd}} = q$.
- On the other hand

$$\begin{cases} W_2 \oplus (\Lambda^2 V^{\text{even}} \cap dV^{\text{odd}}) = \Lambda^2 V^{\text{even}} \\ \dim \Lambda^2 V^{\text{even}} = \dfrac{n(n+1)}{2} \\ \dim(\Lambda^2 V^{\text{even}} \cap dV^{\text{odd}}) \leq n \\ \dim W_2 \geq \dfrac{n(n+1)}{2} - n \end{cases}$$

- Finally $\begin{aligned} \dim H^*(\Lambda V, d) &\geq \tfrac{n(n+1)}{2} + 1 + q \\ &\geq 2n + q \\ &= \dim V \end{aligned}$

- The minimalty of the model $(dV \subset \Lambda^{\geq 2} V)$ insures us that $W_0 \oplus W_1 \oplus W_2 \oplus V_0^{\text{odd}}$ is a direct sum in $H^*(\Lambda V, d)$.
- $\dim W_0 = 1, \dim W_1 = n, \dim V_0^{\text{odd}} = q$.
- On the other hand

$$\begin{cases} W_2 \oplus (\Lambda^2 V^{\text{even}} \cap dV^{\text{odd}}) = \Lambda^2 V^{\text{even}} \\ \dim \Lambda^2 V^{\text{even}} = \frac{n(n+1)}{2} \\ \dim(\Lambda^2 V^{\text{even}} \cap dV^{\text{odd}}) \leq n \\ \dim W_2 \geq \frac{n(n+1)}{2} - n \end{cases}$$

- Finally
$$\begin{aligned} \dim H^*(\Lambda V, d) &\geq \tfrac{n(n+1)}{2} + 1 + q \\ &\geq 2n + q \\ &= \dim V \end{aligned}$$

- The minimalty of the model $(dV \subset \Lambda^{\geq 2} V)$ insures us that $W_0 \oplus W_1 \oplus W_2 \oplus V_0^{odd}$ is a direct sum in $H^*(\Lambda V, d)$.
- $\dim W_0 = 1, \dim W_1 = n, \dim V_0^{odd} = q$.
- On the other hand

$$\begin{cases} W_2 \oplus (\Lambda^2 V^{even} \cap dV^{odd}) = \Lambda^2 V^{even} \\ \dim \Lambda^2 V^{even} = \dfrac{n(n+1)}{2} \\ \dim(\Lambda^2 V^{even} \cap dV^{odd}) \leq n \\ \dim W_2 \geq \dfrac{n(n+1)}{2} - n \end{cases}$$

- Finally $\dim H^*(\Lambda V, d) \geq \frac{n(n+1)}{2} + 1 + q$

$$\begin{aligned} &\geq 2n + q \\ &= \dim V \end{aligned}$$

- The minimalty of the model $(dV \subset \Lambda^{\geq 2} V)$ insures us that $W_0 \oplus W_1 \oplus W_2 \oplus V_0^{\text{odd}}$ is a direct sum in $H^*(\Lambda V, d)$.
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- Finally $\begin{aligned} \dim H^*(\Lambda V, d) &\geq \tfrac{n(n+1)}{2} + 1 + q \\ &\geq 2n + q \\ &= \dim V \end{aligned}$

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Maybe the conjecture H will be resolved in the general case for elliptic spaces

Acknowledgements

It is for me a pleasure to thank the following professors for their interest and for their several readings and corrections.

- Mohamed Rachid Hilali: Univ. Casablanca, Morocco.
- Barry Jessup: Univ. Ottawa, Canada.
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