Homotopie et Homologie
Exercise Set 4

13.10.2011

1. Apply the Seifert-van Kampen Theorem to establishing the following isomorphisms.

   (a) $\pi_1(S^1 \vee \cdots \vee S^1, 1) \cong \mathbb{Z} * \cdots * \mathbb{Z}$.

   (b) $\pi_1(S^n, z_0) \cong \{0\}$ for all $z_0 \in S^n$ and all $n \geq 2$. Why can't we use Seifert -van Kampen to conclude that $\pi_1(S^1, 1)$ is trivial as well?

   (c) If $f : (S^1, 1) \to (X, x_0)$ is a based continuous map, then

   $$\pi_1(X \cup_f D^2, x_0) \cong \pi_1(X, x_0)/N,$$

   where N is the normal subgroup generated by $\big\{[f]_*\big\}$.

2. For any positive integer n, explain how to construct a path-connected space X_n from a circle and a disk such that $\pi_1(X_n, x_0) \cong \mathbb{Z}/n\mathbb{Z}$. Show that X_2 is homeomorphic to $\mathbb{R}P^2$, the *real projective plane*, which is usually defined as the quotient of D^2 by the relation that identifies antipodal points on its boundary.