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Prepas MPSI
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# Corrigé Simulation DS1

## EX-1

1) $\exists y \in \mathbb{R}_+ \quad \forall x \in \mathbb{R},\ y \neq x^2$

2) ~~$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ f(x) \leq y$~~

~~$\forall x \in \mathbb{R},\ \exists y$~~

$\exists x \in \mathbb{R},\ \forall y \in \mathbb{R}\ ,\ f(x) > y$ et $f(x) < y$

3) $\exists \varepsilon > 0 \quad \forall \alpha > 0,\ \exists (x,y) \in \mathbb{R}$ tq $|x-y| < \alpha$ et $|f(x) - f(y)| \geq \varepsilon$

4) $\forall x \in \mathbb{R},\ \forall y \in \mathbb{R},\ \forall z \in \mathbb{R}$

$x < y < z$ et $\left( \begin{array}{c} f(x) \geq f(y) \\ \text{ou} \\ f(y) \leq f(z) \end{array} \right)$

EX2 et EX3 voir corrigé Td ECS1

## EX4

1) $S_{n+1} = \sum_{k=1}^{n+1} \frac{1}{k} \binom{n+1}{k} (-1)^{k-1}$

$S_n = \sum_{k=1}^{n} \frac{1}{k} \binom{n}{k} (-1)^{k-1} = \sum_{k=1}^{n+1} \frac{1}{k} \binom{n}{k} (-1)^{k-1}$

car $\binom{n}{n+1} = 0$ par convention

2) Selon la formule du Triangle de Pascal

$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$

Donc $\binom{n+1}{k} - \binom{n}{k} = \binom{n}{k-1}$

cad $S_{n+1} - S_n = \sum_{k=1}^{n+1} \frac{1}{k}\binom{n}{k-1}(-1)^{k-1}$

chgt indic $p=k-1$

$$= \sum_{p=0}^{n} \frac{1}{p+1}\binom{n}{p}(-1)^p$$

or $(1+t)^n = \sum_{p=0}^{n} \binom{n}{p} t^p$

donc $\int_0^x (1+t)^n \, dt = \sum_{p=0}^{n} \binom{n}{p}\frac{x^{p+1}}{p+1}$

ie $\frac{(1+x)^{n+1}-1}{(n+1)x} = \sum_{p=0}^{n} \frac{1}{p+1}\binom{n}{p}x^p$

pour $x=-1$

$$S_{n+1} - S_n = \sum_{p=0}^{n} \frac{1}{p+1}\binom{n}{p}(-1)^p = \frac{1}{n+1}$$

3) $\frac{1}{k}\binom{n}{k-1} = \frac{1}{k}\frac{n!}{(k-1)!\,(n-k+1)!}$

$$= \frac{n!}{k!\,(n-k+1)!}$$

$\frac{1}{n+1}\binom{n+1}{k} = \frac{(n+1)!}{(n+1)\,k!\,(n+1-k)!}$

$$= \frac{n!}{k!\,(n+1-k)!}$$

d'où égalité

4) voir qst 2

5) $\sum_{k=1}^{n} \frac{1}{k} = \sum_{k=2}^{n} (S_k - S_{k-1}) + S_1$

$$= S_2 - S_1 + S_1$$
$$+ S_3 - S_2$$
$$+ S_n - S_{n-1} = S_n$$