

$$\sum_{k=0}^{n-1} \omega^k = \frac{1-\omega^n}{1-\omega} = \begin{cases} n & \text{si } p \notin n\mathbb{N} \\ 1 & \text{si } p \in n\mathbb{N} \end{cases}$$

b)  $I_{n,p} = \sum_{k=0}^{n-1} (\omega^p)^k = \begin{cases} \frac{1-\omega^{np}}{1-\omega^p} = 0 & \text{si } p \notin n\mathbb{N} \\ n & \text{si } p \in n\mathbb{N} \end{cases}$

c)  $C_n + iS_n = \frac{\sin(a+b(n-\frac{1}{2})) - \sin(a-\frac{b}{2})}{2\sin\frac{b}{2}} + i \frac{\cos(a-\frac{b}{2}) - \cos(a+b(n-\frac{1}{2}))}{2\sin\frac{b}{2}}$

$C_n = \frac{\sin(a+b(n-\frac{1}{2})) - \sin(a-\frac{b}{2})}{2\sin\frac{b}{2}}$  et  $S_n = \frac{\cos(a-\frac{b}{2}) - \cos(a+b(n-\frac{1}{2}))}{2\sin\frac{b}{2}}$

si  $b \neq 0 [2\pi]$  et si  $b \equiv 0 [2\pi]$   $C_n = n \cos a$  et  $S_n = n \sin a$ .

2 -  $f(1) + f(\omega) + f(\omega^2) + f(\omega^3) = \sum_{l=0}^3 (1+\omega^l)^p = \sum_{l=0}^3 \sum_{k=0}^p C_p^k \omega^{kl} = \sum_{k=0}^p C_p^k \sum_{l=0}^3 \omega^{kl}$

3)  $= \sum_{k=0}^p C_p^k (1 + \omega^k + \omega^{2k} + \omega^{3k}) = 4 \sum_{0 \leq 4k \leq p} C_p^{4k}$

car  $1 + \omega^k + \omega^{2k} + \omega^{3k} = \begin{cases} 0 & \text{si } k \notin 4\mathbb{N} \\ 4 & \text{si } k \in 4\mathbb{N} \end{cases}$

d'autre part  $f(1) + f(\omega) + f(\omega^2) + f(\omega^3) = 2^p + (1+i)^p + (1-i)^p = 2^p + 2^{\frac{p}{2}+1} \cos(\frac{p\pi}{4})$

et par suite  $\sum_{0 \leq 4k \leq p} C_p^{4k} = 2^p + 2^{\frac{p}{2}+1} \cos(\frac{p\pi}{4})$

3-a)  $\sum_{k=0}^{n-1} \omega^{(j+k)^2} = \sum_{k=0}^{n-1} \omega^{(nq+r+k)^2} = \sum_{k=0}^{n-1} \omega^{(r+k)^2}$

3)  $= \omega^{r^2} + \omega^{(r+1)^2} + \dots + \omega^{(n-1)^2} + 1 + \omega^2 + \dots + \omega^{(r+n-1)^2}$

$= \omega^{r^2} + \omega^{(r+1)^2} + \dots + \omega^{(n-1)^2} + 1 + \omega^2 + \dots + \omega^{(r-1)^2}$

$= G_n$

$$3 - b) \sum_{k=0}^{n-1} \omega^{k^2} I_{n, 2k} = \sum_{k=0}^{n-1} \omega^{k^2} \sum_{l=0}^{n-1} \omega^{2kl} = \sum_{k=0}^{n-1} \sum_{l=0}^{n-1} \omega^{k^2 + 2kl}$$

$$\textcircled{3} = \sum_{l=0}^{n-1} \omega^{-l^2} \sum_{k=0}^{n-1} \omega^{(k+l)^2} = \sum_{l=0}^{n-1} \overline{\omega^{l^2}} \cdot G_n$$

$$= G_n \left( \sum_{l=0}^{n-1} \omega^{l^2} \right) = G_n \overline{G_n}$$

$$c) |G_n|^2 = G_n \overline{G_n} = \sum_{k=0}^{n-1} \omega^{k^2} I_{n, 2k} = \begin{cases} n(1 + (-1)^{n/2}), & n \text{ st paire} \\ n & \text{si } n \text{ st impaire} \end{cases}$$

$\textcircled{1,5}$

$$\text{car } I_{n, 2k} = \begin{cases} 0 & \text{si } 2k \notin n\mathbb{N} \\ n & \text{si } 2k \in n\mathbb{N} \end{cases}$$

4 - a)

$\textcircled{2,5}$

$$\sum_{k=0}^{n-1} (1 + \omega^k)^p = \sum_{k=0}^{n-1} \sum_{l=0}^p C_p^l \omega^{kl} = \sum_{l=0}^p \sum_{k=0}^{n-1} C_p^l \omega^{kl}$$

$$= \sum_{l=0}^p C_p^l \left( \sum_{k=0}^{n-1} \omega^{kl} \right) = \sum_{l=0}^p C_p^l I_{n, l} = n \sum_{\substack{0 \leq k \leq p \\ k \equiv 0 \pmod{n}}} C_p^k$$

b)

donc pour  $n=4$

$$4 \sum_{0 \leq k \leq p/4} C_p^{4k} = \sum_{k=0}^3 (1 + \omega^k)^p = f(1) + f(\omega) + f(\omega^2) + f(\omega^3)$$

$\textcircled{1,5}$