

DS N°2

Corrigé

Problème :

1. Cours
2. $n \wedge m = 1$
3. $\varphi(n)$
 - a. Facile
 - b. Facile
 - c. $\text{Card}(G) = nm$
4. Par recurrence.
5. $(\alpha, \beta)^k = (\alpha^k, \beta^k) = (1, 1) \Leftrightarrow \alpha^k = 1, \beta^k = 1 \Leftrightarrow o(a)/k, o(b)/k \Leftrightarrow o(a) \vee o(b)/k$
6. car $o(a) \leq n$ et $o(b) \leq m$.
7. G cyclique
 $\Rightarrow \exists (\alpha, \beta) \in G/G = \langle (\alpha, \beta) \rangle \Rightarrow \text{Card}(G) = o((\alpha, \beta)) \Rightarrow nm = o(a) \vee o(b) \leq n \vee m \Rightarrow n \wedge$
 - a. $\text{Card}(\langle (\omega_n, \omega_m) \rangle) = o(\omega_n) \vee o(\omega_m) = n \vee m = nm = \text{Card}(G)$
 - b. (α, β) engendre $G \Leftrightarrow \langle (\alpha, \beta) \rangle = G \Leftrightarrow o((\alpha, \beta)) = \text{Card}(G) \Leftrightarrow o(a) \vee o(b) = nm \Leftrightarrow o(a) = n, o(b) = m$ car $o(a) \leq n, o(b) \leq m$
 - c. D'après b

Exercice : Facile