1 $\quad \sum_{k=0} \quad = \quad \frac{}{1-\omega}$ $\qquad$ 1 $\quad \prod_{k=0} \omega = \omega \quad = \omega^{\frac{n(n-1)}{2}} = \left\{ \begin{array}{l} \\ 1 \text{ si } n \text{ est impair} \end{array} \right.$

b) $$I_{n,p} = \sum_{k=0}^{n-1} (\omega^p)^k = \begin{cases} \dfrac{1-\omega^{np}}{1-\omega^p} = 0 & \text{si } p \notin n\mathbb{N} \\ n & \text{si } p \in n\mathbb{N} \end{cases}$$ 1,5

c) $$c_n + i S_n = \frac{\sin\left(a+b\left(n-\frac{1}{2}\right)\right) - \sin\left(a-\frac{b}{2}\right)}{2\sin b/2} + i\, \frac{\cos\left(a-\frac{b}{2}\right) - \cos\left(a+b\left(n-\frac{1}{2}\right)\right)}{2\sin\frac{b}{2}}$$ 1,5

$$c_n = \frac{\sin\left(a+b\left(n-\frac{1}{2}\right)\right) - \sin\left(a-\frac{b}{2}\right)}{2\sin\frac{b}{2}} \quad \text{et} \quad S_n = \frac{\cos\left(a-\frac{b}{2}\right) - \cos\left(a+b\left(n-\frac{1}{2}\right)\right)}{2\sin\frac{b}{2}}$$ 1

si $b \not\equiv 0\ [2\pi]$ et si $b \equiv 0\ [2\pi]$ $c_n = n\cos a$ et $S_n = n\sin a$.

2 - $$f(1)+f(\omega)+f(\omega^2)+f(\omega^3) = \sum_{l=0}^{3} (1+\omega^l)^p = \sum_{l=0}^{3}\sum_{k=0}^{p} C_p^k \omega^{kl} = \sum_{k=0}^{p} C_p^k \sum_{l=0}^{3} \omega^{kl}$$

3 $$= \sum_{k=0}^{p} C_p^k \left(1+\omega^k+\omega^{2k}+\omega^{3k}\right) = 4 \sum_{0 \le 4k \le p} C_p^{4k}$$

car $$1+\omega^k+\omega^{2k}+\omega^{3k} = \begin{cases} 0 & \text{si } k \notin 4\mathbb{N} \\ 4 & \text{si } k \in 4\mathbb{N} \end{cases}$$

d'autre part $$f(1)+f(\omega)+f(\omega^2)+f(\omega^3) = 2^p + (1+i)^p + (1-i)^p = 2^p + 2^{\frac{p}{2}+1} \cos\left(\frac{p\pi}{4}\right).$$

et par suite $$\sum_{0 \le 4k \le p} C_p^{4k} = \frac{2^p + 2^{p/2+1} \cos\left(\frac{p\pi}{4}\right)}{4}.$$

3 - a) $$\sum_{k=0}^{n-1} \omega^{(j+k)^2} = \sum_{k=0}^{n-1} \omega^{(nq+r+k)^2} = \sum_{k=0}^{n-1} \omega^{(r+k)^2}$$

3 $$= \omega^{r^2} + \omega^{(r+1)^2} + \dots + \omega^{(n-1)^2} + 1 + \omega^{2^2} + \dots + \omega^{(r+n-1)^2}$$

$$= \omega^{r^2} + \omega^{(r+1)^2} + \dots + \omega^{(n-1)^2} + 1 + \omega^{2^2} + \dots + \omega^{(r-1)^2}$$

$$= G_n.$$

3 - b) $\sum_{k=0}^{n-1} \omega^{k^2} I_{n,2k} = \sum_{k=0}^{n-1} \omega^{k^2} \sum_{l=0}^{n-1} \omega^{2kl} = \sum_{k=0}^{n-1}\sum_{l=0}^{n-1} \omega^{k^2+2kl}$

3

$$= \sum_{l=0}^{n-1} \omega^{-l^2} \sum_{k=0}^{n-1} \omega^{(k+l)^2} = \sum_{l=0}^{n-1} \overline{\omega^{l^2}}\, G_n$$

$$= G_n \overline{\left(\sum_{l=0}^{n-1} \omega^{l^2}\right)} = G_n \overline{G_n} .$$

c) $|G_n|^2 = G_n \overline{G_n} = \sum_{k=0}^{n-1} \omega^{k^2} I_{n,2k} = \begin{cases} n(1+(-1)^{n/2}), & n \text{ est paire} \\ n & \text{si } n \text{ est impaire} \end{cases}$

1,5

car $I_{n,2k} = \begin{cases} 0 & \text{si } 2k \notin n\mathbb{N} \\ n & \text{si } 2k \in n\mathbb{N} . \end{cases}$

4 - a) $\sum_{k=0}^{n-1} (1+\omega^k)^p = \sum_{k=0}^{n-1} \sum_{l=0}^{p} C_p^l \omega^{kl} = \sum_{l=0}^{p}\sum_{k=0}^{n-1} C_p^l \omega^{kl}$

2,5

$$= \sum_{l=0}^{p} C_p^l \left(\sum_{k=0}^{n-1} \omega^{kl}\right) = \sum_{l=0}^{p} C_p^l I_{n,l} = n \sum_{0 \leq k \leq p/n} C_p^{kn}$$

b) donc pour $n = 4$ $\quad 4 \sum_{0 \leq k \leq p/4} C_p^{4k} = \sum_{k=0}^{3} (1+\omega^k)^p$

$$= f(1) + f(\omega) + f(\omega^2) + f(\omega^3)$$

1,5