بِسمِ اللَّهِ الرَّحمَنِ الرَّحِيمِ وَ قُلِ إعمَلُوا فَسَيَرَى اللَّهُ عَمَلَكُم وَ رَسُولُهُ وَ المُؤ مِنُون صَدَقَ اللَّهُ العَظِيم

# Corrigé du Contrôle 2: *Récurrence, sommes Limites, continuité*

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## *Exercice 1 .*

1) **La fonction** $x \mapsto E(x)$ **est continue sur** $\mathbb{R}_+ \setminus \mathbb{N}$**, donc** $\mapsto E(x)$ **est continue sur** $\mathbb{R}_+ \setminus \left\{ \dfrac{1}{n},\ \text{tel que } n \in \mathbb{N}^* \right\}$

2) **L'allure de la courbe est en escalier qui descendent de** $+\infty$ **jusqu'à 0.**

## *Exercice 2 .*

1) $\mathcal{C}_n^0 = \dfrac{n!}{n!} = 1, \mathcal{C}_n^1 = \dfrac{n!}{(n-1)!} = n$ **et** $\mathcal{C}_n^2 = \dfrac{n!}{(n-2)!2!} = \dfrac{n(n-1)}{2}$.

2) **Trés évident**

3) $\mathcal{C}_n^p + \mathcal{C}_n^{p+1} = \dfrac{n!}{p!(n-p)!} + \dfrac{n!}{(p+1)!(n-p-1)!} = \dfrac{n!(p+1)}{(p+1)!(n-p)!} + \dfrac{(n-p)n!}{(p+1)!(n-p)!} = \dfrac{n!(n+1)}{(p+1)!(n-p)!} = \dfrac{(n+1)!}{(p+1)!(n-p)!} = \mathcal{C}_{n+1}^{p+1}$.

4) **Le résultat est vrai pour** $n = 0$.

**Supposons que** $(a+b)^n = \sum_{p=0}^{n} \mathcal{C}_n^p a^p b^{n-p}$**, donc**

$$\begin{aligned}
(a+b)^{n+1} &= (a+b)\left(\sum_{p=0}^{n} \mathcal{C}_n^p a^p b^{n-p}\right) = \sum_{p=0}^{n} \mathcal{C}_n^p a^{p+1} b^{n-p} + \sum_{p=0}^{n} \mathcal{C}_n^p a^p b^{n-p+1} \\
&= \sum_{p=1}^{n+1} \mathcal{C}_n^{p-1} a^p b^{n-p+1} + \sum_{p=0}^{n} \mathcal{C}_n^{p-1} a^p b^{n-p+1} = \sum_{p=1}^{n} (\mathcal{C}_n^{p-1} + \mathcal{C}_n^p) a^p b^{n-p+1} + a^{n+1} + b^{n+1} \\
&= \sum_{p=1}^{n} \mathcal{C}_{n+1}^p a^p b^{n+1-p} + a^{n+1} + b^{n+1} = \sum_{p=0}^{n+1} \mathcal{C}_{n+1}^p a^p b^{n+1-p}
\end{aligned}.$$

5) a) $\sum_{p=0}^{n} \mathcal{C}_n^p = 2^n$**, prendre** $a = b = 1$.

b) $\sum_{p=0}^{n} \mathcal{C}_n^p (-1)^p = 0$**, prendre** $a = -1, b = 0$.

c) **D'abord on remarque que** $S = \sum_{p=0}^{n} p\mathcal{C}_n^p = \sum_{p=1}^{n} p\mathcal{C}_n^p$**, d'autre part** $p\mathcal{C}_n^p = \dfrac{n!p}{p!(n-p)!} = n\dfrac{(n-1)!}{(p-1)!(n-p)!} = n\mathcal{C}_{n-1}^{p-1}$**, d'où** $S = n\sum_{p=1}^{n} \mathcal{C}_{n-1}^{p-1} = n\sum_{p=0}^{n-1} \mathcal{C}_{n-1}^p = n2^{n-1}$.

***Fin***
***à la prochaine***