MPSI 1 **2002-2003**

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Corrigé

Exercice 1:

1. $p = 0 \Rightarrow S_n = n \to +\infty, p = 1 \Rightarrow S_n = \sum_{k=1}^{n} \frac{1}{k} \to +\infty$ (vu en TD)
2. facile utiliser la definition : $C_n^k = \frac{n!}{k!(n-k)!}$
3. Par recurrence sur n : $(p-1)S_{n+1} = (p-1)S_n + (p-1)u_{n+1} = 1 - (n+p+1)u_{n+1} + (p-1)u_{n+1} = 1 - (n+2)u_{n+1} =$
4. $\frac{v_{n+2}}{v_{n+1}} = \frac{(n+p+2)u_{n+2}}{(n+p+1)u_{n+1}} = \frac{n+2}{n+p+1} \leq 1$
5. car decroissante minorée par 0
6. $\frac{1}{u_n} = \frac{(n+p)!}{n!p!} = \frac{1}{p!}(n+1)(n+2).....(n+p) \geq \frac{n}{p!} \to +\infty$ (*)
7. $(p-1)S_n = 1 - (n+p)u_{n+1} + u_{n+1} \to 1 - l = (p-1)S$
8. d'aprés (*) $\frac{1}{v_n} = \frac{1}{(n+p)u_n} \geq \frac{n}{p!} \to +\infty, S = \frac{1}{p-1}$

Exercice 2:

1. $nr^n = \frac{n}{a^n} \to +\infty, a = \frac{1}{r}$
2. $\ln(n)r^n = \frac{\ln(n)}{a^n} \to +\infty, a = \frac{1}{r}$
3. $\int_k^{k+1} \frac{1}{t}dt \leq \frac{1}{k} \leq \int_{k-1}^{k} \frac{1}{t}dt \Rightarrow \int_2^{n+1} \frac{1}{t}dt = \ln(n+1) - \ln(2) = \ln(n)\ln(1+\frac{1}{n}) - \ln(2) \leq \sum_{k=2}^{n} \frac{1}{k}$